


Research Article

Reverse Engineering for Real Tooth Surfaces of Generated Spiral Bevel Gears

Liu Guanglei*, Wang Leyun, Li Degeng, Chang Kai

Abstract

The manufacturing parameters of spiral bevel gear should be reversed first in reverse engineering of spiral bevel gear drives. An approach is proposed to reconstruct the real tooth surfaces of generated spiral bevel gears based on tooth surface grids formed by a coordinate measuring machine (CMM) without knowing the design method of the gear flank and actual manufacturing parameters. The trial tooth surface grid (TTSG), either the concave side or the convex side, represented by the manufacturing parameters of duplex method, are formed by solving a set of non-linear equations based on the CMM tooth surface grid (CTSG) in the projection plane. When the CTSG is rotated until its center node coincided with that the TTSG, the tooth surface deviation is formed. A unified objective for optimization, which consists of the tooth surface deviations of both the concave and the convex sides, is set up. The design variables are the blade pressure angle and the point radius, both for the outer blade and the inner blade in addition to the spiral angle. The constraints are the changing manufacturing parameters and the dedendum angle which are dependent on the design variables. A hybrid strategy for reverse engineering, which consists of the discrete optimization and the continuous optimization, is employed in turn. A pair of spiral bevel gear drives in aviation engine is explored. The results show that: (i) the reversed head cutter diameter is smaller than the one with the least shrinkage ratio of the pinion root slot, indicating that the prototype designer pays more attention on increasing the fatigue strength and decreasing the sensibility of tooth contact pattern due to misalignments; (ii) the pressure angle of the outer blade and the inner blade are almost the same, meaning the pressure angle of the blades are not modified; (iii) the dedendum angle related to tooth taper types is calculated according the addendum or dedendum of the pinion over the whole tooth depth.

Keywords: Spiral bevel gears; Tooth taper; Manufacturing parameters; Tooth surface reconstruction; Reverse engineering

Introduction

In the exploration of manufacturing parameters of spiral bevel gears with satisfying meshing performance, an iteration process from design through machining, assemblage and application to re-design is required. The tooth surfaces settled down are called theoretical tooth surfaces, which will be used as a reference to judge the accuracy level of actual tooth surfaces. The corresponding manufacturing parameters are thus called theoretical

Affiliation:

School of Mechatronics, Northwestern Polytechnical University, Xi'an, 710072, China

*Corresponding author:

LIU Guanglei, School of Mechatronics, Northwestern Polytechnical University, Xi'an, 710072, China.

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manufacturing parameters. Nowadays, most spiral bevel gears are produced on numerical controlled machines. This ensures that the spiral bevel gears meet the design specifications with sufficient accuracy. However, how to prescribe the theoretical tooth surfaces still confuses the designers and the manufacturers alike. Reverse engineering of spiral bevel gears is the very expertise. By applying the technique of reverse engineering to spiral bevel gears, the theoretical tooth surfaces and the corresponding manufacturing parameters can be determined in a short time. Engineers can take advantage of this expertise in shortening the development cycle, as well as improving the meshing performance of spiral bevel gears. A great deal of research work has been dedicated to spiral bevel gears. The Gleason Works proposed local conjugate method to design manufacturing parameters [1-2]. Litvin [3-8] put forward local synthesis to actively control the meshing performances at the mean contact point. Sheveleva [9] transformed the tooth surfaces into grids on a tangent plane at the mean contact point by means of projection and performed the tooth contact analysis based on the projected grids. Krenzer [10] proposed a method to ascertain errors of the actual manufacturing parameters. Gosselin et al. [11-13] developed a methodology to identify the actual manufacturing parameters. Shunmugam, Rao, and Jayaprakash [14] established a mathematical model to construct the normal deviation between the actual tooth surface and the theoretical tooth surface. Litvin [15] set up an over-determined equation relating the deviation of the actual tooth surface with respect to the theoretical tooth surface. By solving the equation, the errors of the actual machine-tool settings were identified. The above researches related to finding the actual machine-tool settings have one thing in common. They conducted the research work under the condition of having already known the theoretical manufacturing parameters and the corresponding tooth surfaces. However, strict sense of reverse engineering of spiral bevel gears does not have such advantages. Engineers who apply reversing engineering to spiral bevel gears are asked to uncover the actual manufacturing parameters without having known the theoretical manufacturing parameters in advance. All they have in hand is the tooth blank. It is reasonable that they take spiral bevel gears used in well-done products as prototype gears, and the tooth surfaces of the gear as the theoretical tooth surfaces because these spiral bevel gears have already been produced with sufficient accuracy. Thus the ultimate goal of reversing spiral bevel gears is to reconstruct the actual tooth surfaces of the pinion and the gear, respectively. Spiral bevel gears are engaged in point contact. The gear of spiral bevel gear drives is usually produced first. And the pinion is then produced to match the gear. Therefore the reversing techniques of the gear in a spiral bevel gear pairs should be explored first. This paper aims at reconstructing the theoretical tooth surface of the gear in a spiral bevel gear pairs. The basic

idea is to minimize the deviation between the CMM tooth surface grids (CTSG) and the trial tooth surface grids (TTSG) expressed in manufacturing parameters of duplex method. By fine-tuning the manufacturing parameters, the TTSG, both concave side and convex side, approaches to the CTSG. The tooth surfaces of the gear flank and the corresponding manufacturing parameters are thus reconstructed.

Tooth Surface of Generated Gear

Spiral bevel gears are usually milled or grinded by duplex method without modified roll motion. The tooth surfaces of spiral bevel gears are closely related with the processing method and machine-tool settings. When the pitch angle is less than 70° , the gear is generated. Otherwise, the gear is formatted in order to raise productivity. When the gear is formatted, helical motion can be additionally employed to improve the conjugate condition. This paper is limited to the reversing techniques of the generated spiral bevel gears. When the spiral bevel gear is generated, the tooth surfaces are dependant on tooth taper type, cutter parameters and machine-tool settings.

Tooth taper type

In the process of generating the gear tooth surface by the duplex method, the root cone is expected to be cut out at the same time. Thereafter the machine-tool installation angle of the gear must be equal to the root angle γ_2 , as in Figure 1. The root angle equals to the pitch angle minus the dedendum angle as follow:

$$\gamma_2 = \delta_2 - \theta_{r2}, \quad (1)$$

where δ_2 is pitch angle, θ_{r2} is dedendum angle.

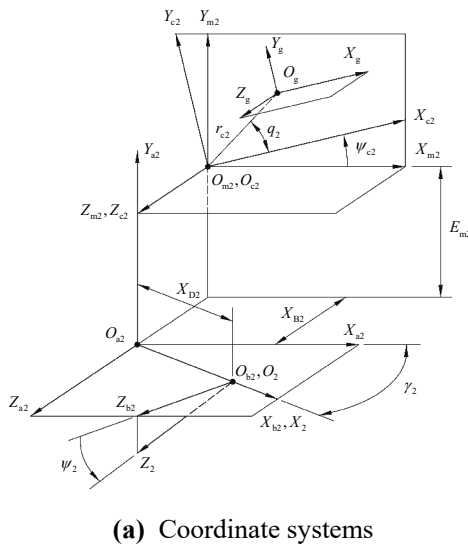
The dedendum angle is related to tooth taper type and spiral angle according to the mechanism of dedendum angle calculation. When the sum of dedendum angles is proportionally distributed among the pinion and gear, eight tooth taper types can be employed by different combinations of the height of addendum or dedendum of the pinion or gear over the whole depth.

Machine-tool settings of gear

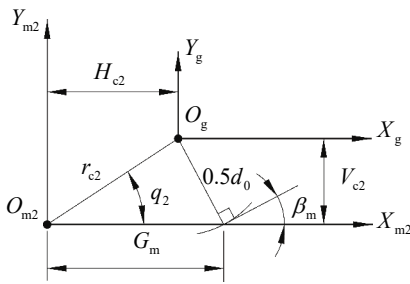
The machine-tool settings are illustrated in Figure 1. They are the radial setting r_{c2} , installment angle q_2 , the blank offset E_{m2} , the sliding base X_{B2} , the machine center to back X_{D2} , and the installment angle γ_2 . These parameters are only related to blank parameters. The blank offset E_{m2} and machine center to back X_{D2} are both taken to be zeros. ψ_{c2} and ψ_2 are the cradle angle and the rotational angle of the gear as in Figure 1. The ratio of cutting is held constant in generation of the gear tooth surfaces.

Equations of cutter surfaces

Most frequently head cutters with straight blades are used



(a) Coordinate systems



(b) Head cutter position and cradle angle

Figure 1: Machine-tool settings in gear generation.

in manufacturing the spiral bevel gear. The revolution of the cutter blades forms the surface of a cone as in Figure 2. It functions like the tooth surface of the imaginary cradle gear. In the coordinate system of cutter S_g , the tooth surface of the cradle and its unit normal is represented as follows:

$$\mathbf{r}_g(u_g, \theta_g) = \begin{bmatrix} (R_{c2} \pm u_g \sin \alpha_g) \cos \theta_g \\ (R_{c2} \pm u_g \sin \alpha_g) \sin \theta_g \\ -s_g \cos \alpha_g \end{bmatrix} \quad (2)$$

$$\mathbf{n}_g(\theta_g) = \begin{bmatrix} \cos \alpha_g \cos \theta_g \\ \cos \alpha_g \sin \theta_g \\ \pm \sin \alpha_g \end{bmatrix} \quad (3)$$

where μ_g and θ_g are taken as the tooth surface parameters, α_g is the pressure angle, R_{c2} is the point radius of the head cutter. The positive sign in “ \pm ” is used to the outer cone or the concave side of the gear, while the negative sign is used to the inner cone or the convex side of the gear.

Gear tooth surface

The envelope to the family of the tooth surfaces of the cradle forms the tooth surface of the gear:

$$\begin{cases} \mathbf{r}_2(u_g, \theta_g, \psi_{c2}) = \mathbf{M}_{2g}(\psi_{c2}) \mathbf{r}_g(u_g, \theta_g) \\ \mathbf{n}_2(\theta_g, \psi_{c2}) = \mathbf{M}_{2g} \mathbf{n}_g(\theta_g) \\ f(u_g, \theta_g, \psi_{c2}) = 0 \end{cases} \quad (4)$$

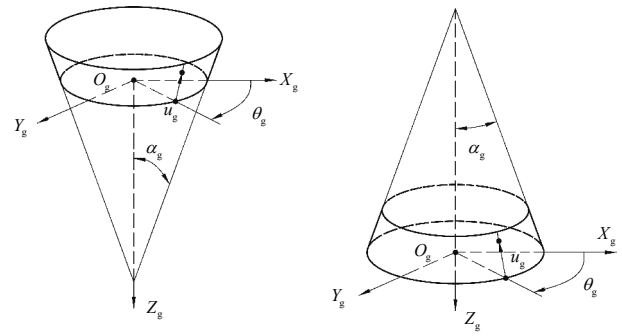


Figure 2: Cone surfaces of the outer and inner blades.

where \mathbf{M}_{2g} and \mathbf{M}'_{2g} are the transformation matrix for position vectors and unit normal vectors from the coordinate system of cutter S_g to the coordinate system of gear S_2 , respectively; $f(u_g, \theta_g, \psi_{c2}) = 0$ is the meshing equation.

Objective for Reversing Tooth Surface

2D grid from CTSG

The gear tooth surfaces are measured by CMM. A grid of 5×9 nodes is formed, in which 5 nodes are along the tooth profile and 9 nodes are along the tooth width. The position vector $\mathbf{r}'_2 = [x'_2 \ y'_2 \ z'_2]^T$ of CTSG is transformed to the projection plane by the following formula:

$$\begin{cases} X_2 = x'_2 \\ R_2 = \sqrt{y'^2_2 + z'^2_2} \end{cases} \quad (5)$$

where X_2 is along the gear axis of rotation; and R_2 is along the radial direction of the gear.

Formation of TTSG

Based on the 2D grid above, a set of nonlinear equations can be formed as follow:

$$\begin{cases} x_2(\theta_g, \psi_{c2}) = X_2 \\ \sqrt{[y_2(\theta_g, \psi_{c2})]^2 + [z_2(\theta_g, \psi_{c2})]^2} = R_2 \\ f(u_g, \theta_g, \psi_{c2}) = 0 \end{cases} \quad (6)$$

where x_2 , y_2 and z_2 are the three components of a position vector $\mathbf{r}_2 = [x_2 \ y_2 \ z_2]^T$ of TTSG.

After the equations above are solved, the TTSG with its nodes corresponding to the CTSG is obtained.

Optimization objective

The center node of the TTSG is coincided with that of the CTSG by rotating operation, and the tooth surface deviation between them is formed. Because the gear tooth surfaces are manufactured by duplex method, the concave side and the convex side are milled or grinded at the same time. Therefore, a unified optimization method is applied. This method combines the tooth surface deviation of the concave and convex side of the gear as one optimization objective as

follow:

$$\begin{cases} \min F(\mathbf{x}) = \sum_{j=1}^2 w_j \sum_{i=1}^{45} |r_{2ij} - r'_{2ij}| \\ \text{s.t. } A(\mathbf{x}) \\ B(\mathbf{x}) \end{cases} \quad (7)$$

where $\mathbf{x} = [\beta_m \ \alpha_g^e \ R_{c_2}^e \ \alpha_g^i \ R_{c_2}^i]^\top$ is the vector of the design variables, in which β_m is the spiral angle, α_g^e and α_g^i are the pressure angles of the outer and inner blade, $R_{c_2}^e$ and $R_{c_2}^i$ are the point radii of the outer and inner blade; w_j ($j=1, 2$) is the weighting coefficient of the concave side or the convex side; the low index i ($i=1, 2, \dots, 45$) represents the sequential node number of the CTSG or the TTSG; $A(\mathbf{x})$ represents the formula of the changing dedendum angle; $B(\mathbf{x})$ represents the equations of the changing machine-tool settings.

Reversing Strategy

Initial values of design variables

The spiral angle is measured from the blank and is taken directly as the initial value. When an imaginary cradle bevel gear with 90° face angle is applied, the pressure angle of the inner blade should be greater than that of the outer blade so that the pressure angle on pitch cone is equal to the nominal pressure angle [17]. The nominal pressure angle can be taken as the initial value in reversing the manufacturing parameters of the gear. The point radius is equal to the head cutter diameter plus or minus half of the cutter width with respect to the outer or inner blade. The point radius is less influenced by the cutter width than the head cutter diameter which is subjected to tooth taper types. Several candidate head cutters with different head cutter diameters may be used for the same tooth taper type. In reversing the gear tooth surfaces, both the tooth taper type of the proto-typed gear and the designer's inclination in selecting the head cutter are unknown. Based on experiences, the initial head cutter can be selected from the head cutter library of Gleason Works according to the principle of the least shrinkage ratio of the pinion root slot. After the head cutter is chosen, the initial point radius can be calculated [16-17].

Discrete optimization for reversing the head cutter diameter

The actual head cutter may not be the one which ensures the least shrinkage of the pinion root slot. This could be explained as follows. A smaller head cutter is helpful to increase the fatigue strength of the gear and reduce the sensitivity of the tooth contact pattern due to misalignments; a bigger head cutter is beneficial to prolong the cutter service life and enhance the productivity. By applying the candidate head cutters around the head cutter with the least shrinkage of the pinion root slot one by one to the optimization objective, the tooth surface deviations for the candidate head cutters are obtained. Obviously, the smallest value corresponds to the

actual head cutter. With this result, the designer's inclination in choosing the head cutter is exposed. This reversing process is called discrete optimization.

Continuous optimization for reversing the tooth surfaces

Having determined the diameter of the head cutter, we can try to find the actual tooth surfaces of the concave side and the convex side simultaneously. By correcting the design variables continuously, the TTSG approaches to the CTSG. When the value of the optimization objective is within the predetermined convergence tolerance, the actual tooth surface grids are considered to be found, as well as the corresponding manufacturing parameters. Since the dedendum angle is related to the tooth taper types, the above optimization process must be carried out for each of the eight different tooth taper types. Apparently, the prototype spiral bevel gear has the smallest value among the eight tooth taper types.

Example

The gear of a pair of spiral bevel gears used in aviation engine is considered to verify the proposed reversing technique.

Initial manufacturing parameters

The basic parameters of the spiral bevel gear pair are shown in Table 1 by measurement and calculation. The initial diameter of the head cutter with the least shrinkage ratio of the pinion root slot and initial pressure angle of cutter blades are calculated and shown in Table 2. The initial machine-tool settings of the gear are listed in Table 3. The CMM tooth surfaces represented by CTSG are drawn in Figure 3.

Discrete optimization for reverse engineering of the head cutter diameter

Five candidate head cutters around the initial head cutter with the least shrinkage ratio of the pinion root slot are

Table 1: Basic blank parameters

Item	Gear
Number of teeth	65
Hand of spiral	RH
Mean spiral angle/(°)	30
Pressure angle/(°)	22.5

Table 2: Initial head cutter data

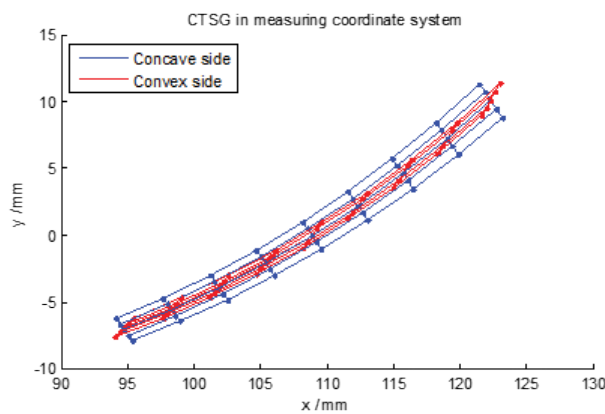
Item	Inner/Outer blade
Diameter /mm	228.6
Pressure angle/(°)	22.5
Cutter width /mm	2.2916

grouped and listed in Table 5. The weighting coefficients for the concave side and convex side in the unified objective function are both taken 0.5 to show that they have the same importance. The objective function is calculated for every head cutter as in Figure 4. In order to show the deviations more clearly, the tooth surface deviations are enlarged by 10 times in the figure.

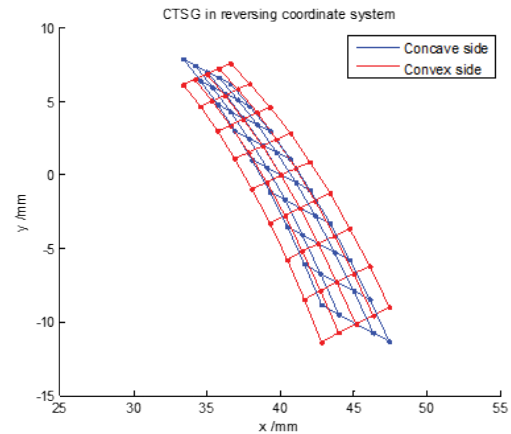
It can be seen from Figure 4 that head cutter 2 has the smallest value of the tooth surface deviation. Thus head cutter 2 is the actual cutter. The actual head cutter is smaller than the initial head cutter which is head cutter 3. It indicates that

Table 3: Initial gear machine-tool settings.

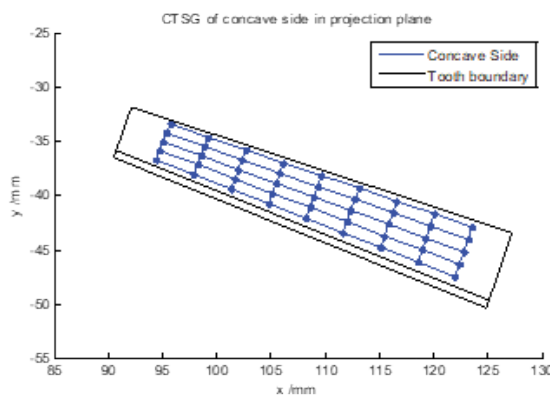
Item	Gear
Radial setting /mm	115.0784
Installment angle / (°)	59.3353
Blank offset /mm	0
Machine center to back /mm	0
Sliding base /mm	0.0509
Ratio of cutting	1.0597



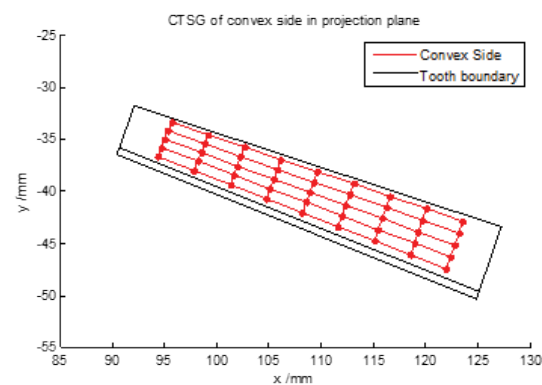
(a) CTSG in measuring coordinate system



(b) CTSG in reversing coordinate system



(c) CTSG of concave side in projection plane



(d) CTSG of convex side in projection plane

Figure 3: CTSG in 2D and 3D.

Table 4: Candidate head cutters/mm.

Head cutter No.	Diameter/mm
1	152.4
2	190.5
3	228.6
4	304.8
5	355.6

the designer preferred to increase the fatigue strength of gear and reduce the sensitivity of the tooth contact pattern due to misalignments.

Continuous optimization for reverse engineering of machine-tool settings

Based on results of the discrete optimization for head cutter diameters, the optimization objective function is then calculated for the eight different tooth taper types. Among them, the specific tooth taper type which is the height of the addendum or dedendum over whole tooth depth in dedendum angle calculation is selected since this tooth taper type has the smallest value of the optimization objective function. The reversing process of the tooth surface deviations for

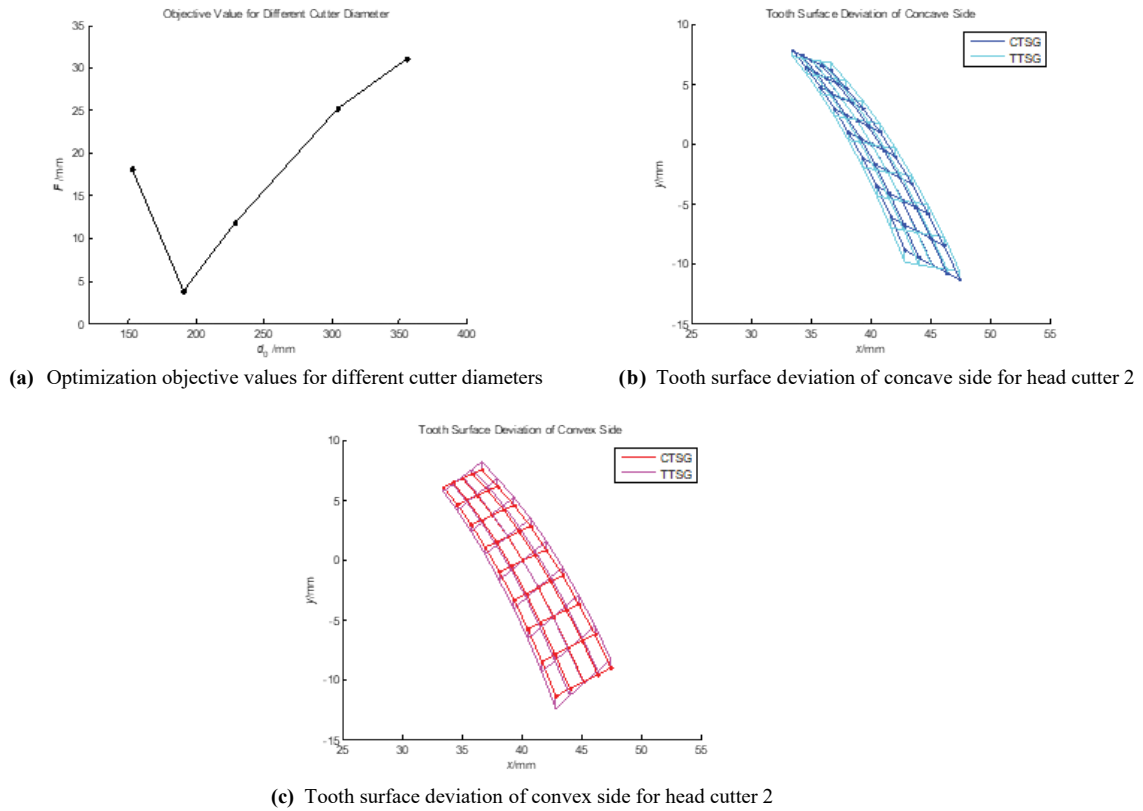


Figure 4: Reversing cutter diameter for convex side

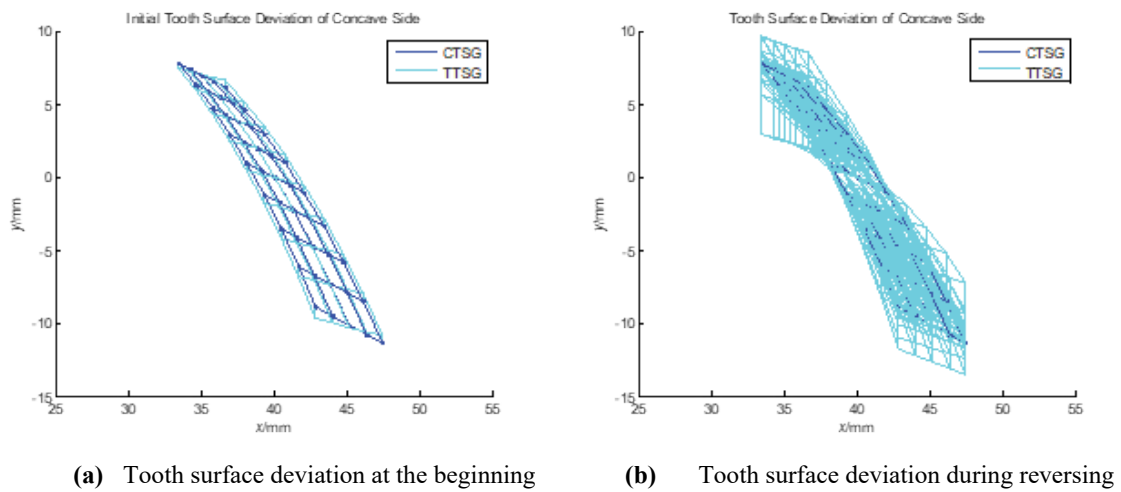
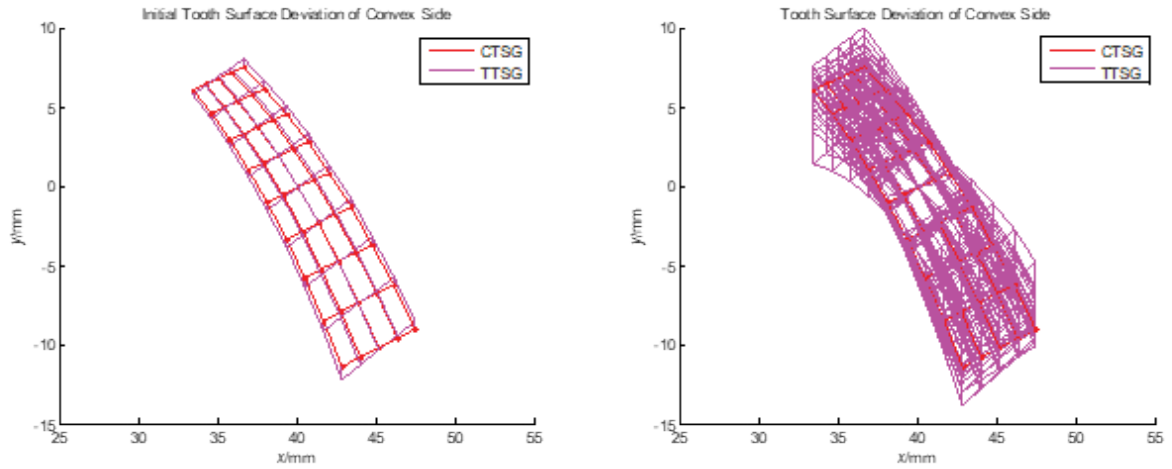
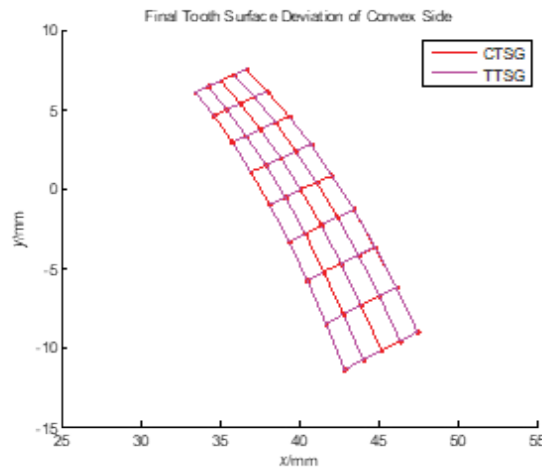


Figure 5: Reversing process for the concave side



(a) Tooth surface deviation at the beginning

(b) Tooth surface deviation during reversing



(c) Tooth surface deviation in the end

Figure 6: Reversing process for the convex side

Table 5: Tooth surface deviation of the concave side for all nodes before reversing /mm.

	1	2	3	4	5
1	0.0329	0.0130	0.0070	0.0272	0.0477
2	0.0367	0.0155	0.0057	0.0272	0.0488
3	0.0408	0.0184	0.0041	0.0269	0.0499
4	0.0454	0.0217	0.0022	0.0264	0.0507
5	0.0505	0.0253	0.0000	0.0255	0.0514
6	0.0559	0.0294	0.0083	0.0244	0.0516
7	0.0621	0.0339	0.0056	0.0229	0.0517
8	0.0687	0.0341	0.0091	0.0210	0.0513
9	0.0760	0.0447	0.0131	0.0186	0.0506

Table 6: Tooth surface deviation of the concave side for all nodes after reversing /mm.

	1	2	3	4	5
1	0.0004	0.0005	0.0004	0.0005	0.0005
2	0.0002	0.0002	0.0002	0.0002	0.0003
3	0.0000	0.0000	0.0001	0.0001	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0001
6	0.0001	0.0002	0.0058	0.0001	0.0001
7	0.0005	0.0004	0.0004	0.0004	0.0003
8	0.0009	0.0039	0.0008	0.0008	0.0008
9	0.0014	0.0015	0.0014	0.0014	0.0014

Table 7: Tooth surface deviation of the convex side for all nodes before reversing /mm.

	1	2	3	4	5
1	0.0371	0.0175	0.0023	0.0224	0.0427
2	0.0401	0.0193	0.0018	0.0231	0.0447
3	0.0433	0.0212	0.0012	0.0239	0.0468
4	0.0465	0.0231	0.0006	0.0246	0.0489
5	0.0500	0.0251	0.0000	0.0255	0.0512
6	0.0536	0.0273	0.0006	0.0263	0.0535
7	0.0574	0.0296	0.0013	0.0271	0.0559
8	0.0615	0.0319	0.0021	0.0281	0.0585
9	0.0658	0.0344	0.0028	0.0290	0.0611

Table 8: Tooth surface deviation of the convex side for all nodes after reversing /mm.

	1	2	3	4	5
1	0.0001	0.0001	0.0000	0.0001	0.0000
2	0.0001	0.0001	0.0001	0.0000	0.0000
3	0.0001	0.0000	0.0000	0.0001	0.0000
4	0.0001	0.0001	0.0000	0.0000	0.0000
5	0.0001	0.0000	0.0000	0.0000	0.0001
6	0.0001	0.0000	0.0001	0.0001	0.0001
7	0.0000	0.0000	0.0001	0.0000	0.0001
8	0.0000	0.0001	0.0000	0.0001	0.0002
9	0.0000	0.0001	0.0002	0.0002	0.0003

Table 9: Reversed head cutter and spiral angle.

Item	Result
Diameter/mm	190.5
Spiral angle/(°)	29.9777

Table 10: Reversed head cutter data.

Item	Inner blade	Outer blade
Nominal diameter /mm	190.5	190.5
Pressure angle /(°)	22.4905	22.5124
Point radius /mm	94.1143	96.3358

Table 11: Reversed gear machine-tool settings.

Item	Gear
Radial setting /mm	107.1021
Installment angle / (°)	50.3865
Blank offset /mm	0.0000
Machine center to back /mm	0.0000
Sliding base /mm	-1.1500
Ratio of cutting	1.0601

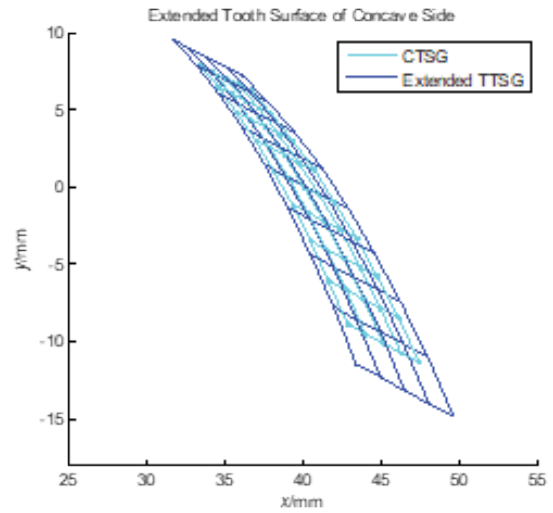


Figure 7: Reconstructed concave side.

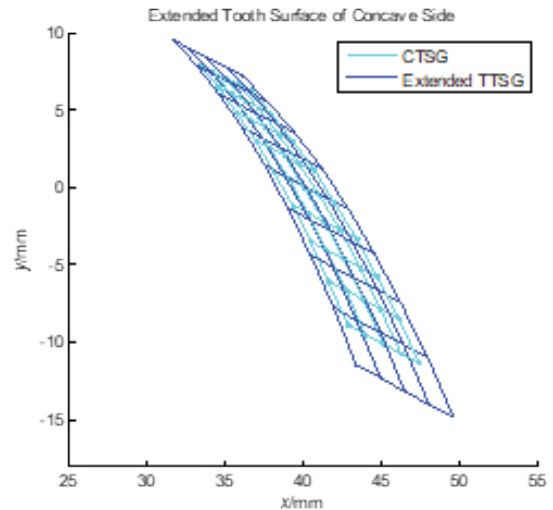


Figure 8: Reconstructed convex side.

this tooth taper type is shown in Figure 5 and Figure 6. The built-in optimization program, `fminsearch`, in MATLAB is used. The convergence precision of the design variables and the optimization objective function are both set to 10^{-3} . The tooth surfaces of the spiral bevel gears are impossible to be expressed in analytical formulae. In order to show the optimization results quantitatively, the values of the tooth surface deviations for all nodes are shown in Table 5~8. The deviations at most nodes are less than $1.5 \mu\text{m}$ as shown in Table 5~8. It meets the requirements of manufacturing accuracy of tooth surfaces. There is only one node on the concave side with deviation of nearly $6 \mu\text{m}$. This could be due to the unpredicted occasionality in the process of measurement. On the whole, the reversed TTSG is quite close to the CTSG. This means that the proposed approach is effective and feasible. The reversed blank data and machine-tool settings are shown in Table 9~11. By contrasting to the data in Table

1~3, there is very small difference between the reversed spiral angle and its nominal value. The difference may be caused by positioning error and elastic deformation of fixtures. There is also very small difference between the pressure angle of the outer blade and that of the inner blade. This indicates that the pressure angle is not corrected and the difference may be caused by errors such as the blade grinding, installation and blade worn-out.

Reconstruction of tooth surfaces

The concave and convex of the gear are reconstructed according to the reversed machine-tool settings as shown in Figure 7 and Figure 8. The grid with dots is the CTSG, the bigger grid with thick lines is the reversed tooth surface.

Conclusions

- (1) Reverse engineering is applied in seeking the manufacture parameters of generated gear of spiral bevel gears and reconstructing the tooth surfaces based on CMM tooth surface data. A pair of spiral bevel gear drives in aviation engine is investigated.
- (2) An initial diameter of the head cutter is found according to the least shrinkage of the pinion root slot. However, the actual head cutter is smaller than the initial head cutter by means of discrete optimization in reverse engineering of the head cutter diameters. It indicates that the prototype designer pays more attention on increasing the fatigue strength and decreasing the sensibility of tooth contact pattern due to misalignments.
- (3) The actual generated tooth surfaces, which are manufactured by duplex method, are reversed for the concave and convex sides simultaneously by correcting the machine-tool settings and the blank parameters according to the optimization objective function. The result indicates that the pressure angle of the outer blade and the inner blade are almost the same, meaning that the pressure angles of the blades are not corrected.
- (4) The optimization objective function for different tooth taper types is calculated. The result indicates that the dedendum angle is calculated according the addendum or dedendum of the pinion over the whole tooth depth in proportional distribution of the sum of dedendum angle.

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